

## Confusing conventional notation and more

There is a wild **mess** in conventional **notation** of axiomatic set theory with letters (Latin, Greek and even Hebrew, small and capital, in various styles) and ad-hoc introduced symbols. Similar expressions may relate to completely different categories. Sometimes the same expressions are used with different meaning so that they can only be understood in local context. Some entities are just introduced metalingually without proper representation in object-language. And one should always be skeptical when 'virtual' entities come into play as it is the case in connection with classes. This mess makes it very complicated for the simple-minded reader and perhaps even hides problems. Axiomatic set-theory talks about well-ordering, but it certainly needs **ORDER** itself!

Conventional publications of axiomatic set theory lack a uniform syntax. Characters are taken from Latin, Greek, Hebrew - small and capital letters, even with special fonts, special symbols and joining them without general rules, not distinguishing between individuals, functions and relations, and worse: mixing object-language and metalanguage. Principle shortcoming is the missing difference between **scheme** and **function-constant**, **formula** and **relation-constant** strings (the **name** of a function or relation and their **value expressions**, often called 'terms'). Some are just given English names that refer to them, and some of these names are not even unique. Finally the reader has to know if something is a **set** or a **class** (that are not really ontological parts).

Conventional mess **Funcish** with uniform, unique and context-independent syntax

<i>sets</i>	$a \dots z \quad a_1 a_2 a_3 \dots$	<b>sort</b>	<b>individual-variable</b>
	$A \dots Z \quad Z_1 Z_2 Z_3 \dots$	$\sigma$	$\sigma_1 \sigma_2 \sigma_3 \dots$
<i>ordinals</i>	$\alpha \beta \gamma \delta \dots \psi$	?	???

$0 = \emptyset = \{\}$	<i>three possibilities</i>	<b>individual-constant</b>
$1 = \{\{\}, \{\{\}\}\}$		$\sigma_n$
$2 = \{\{\}, \{\{\}\}; \{\{\}, \{\{\}\}\}\} \dots$		$\sigma_u = (' \sigma_n)$
$\omega = \aleph_0$	<i>two possibilities</i>	$\sigma_b = ((' \sigma_n)) \dots$
		$\sigma_{vnl}$

	<b>function-constant</b>	<b>scheme</b>	<i>particular-notation</i>
$\{a\}$	$( \sigma)$	$( \sigma_1)$	
$\{a,b\}$	$(\sigma \sigma)$	$(\sigma_1 \sigma_2)$	
$\{a,\{a\}\}$	$(' \sigma)$	$(' \sigma_1)$	
$\{a,b,c\}$	$(\sigma \sigma \sigma)$	$(\sigma_1 \sigma_2 \sigma_3)$	
$\cup a$	$(\cup \sigma)$	$(\cup \sigma_1)$	
$a \cup b$	$(\sigma \cup \sigma)$	$(\sigma_1 \cup \sigma_2)$	
$\cap a$	$(\cap \sigma)$	$(\cap \sigma_1)$	
$a \cap b$	$\sigma \cap \sigma$	$(\sigma_1 \cap \sigma_2)$	
$a/b$	$(\sigma_1/\sigma_2)$	$(\sigma/\sigma)$	
$\langle a,b \rangle$	$(\sigma - \sigma)$	$(\sigma_1 - \sigma_2)$	
$a \times b$	$(\sigma \times \sigma)$	$(\sigma_1 \times \sigma_2)$	
$a^2 \quad a^3$	$(\sigma \times) \quad (\sigma \times \times)$	$(\sigma_1 \times) \quad (\sigma_1 \times \times)$	
$\mathcal{P}a$	$(\uparrow \sigma)$	$(\uparrow \sigma_1)$	

$a \subset b$	$\sigma \subset \sigma$	$\sigma_1 \subset \sigma_2$	<i>particular-notation</i>
$a \subseteq b$	$\sigma \subseteq \sigma$	$\sigma_1 \subseteq \sigma_2$	"
$x < y$	$\sigma < \sigma$	$\sigma_1 < \sigma_2$	"
$x A y \quad abc$	$A(\sigma; \sigma)$	$A(\sigma_1; \sigma_2)$	<i>standard-notation</i>
$f: a \rightarrow b$	$UFU(\sigma; \sigma; \sigma)$	$UFU(\sigma_1; \sigma_2; \sigma_3)$	"

<i><sup>a</sup>b set ? class</i> of mapping sets from <i>a</i> to <i>b</i>	UFUS( $\sigma$ ; $\sigma$ )	UFUS( $\sigma_1$ ; $\sigma_2$ )	"
<i>transitive X</i>	<i>transitivity</i> TR( $\sigma$ )	TR( $\sigma_1$ )	"
<i>well-order X</i>	<i>fundamentality</i> FU( $\sigma$ )	FU( $\sigma_1$ )	"
<i>total-order X</i>	<i>totality</i> TO( $\sigma$ )	TO( $\sigma_1$ )	"
<i>On(X) or <math>X \in On</math></i>	<i>ordinality class</i> OR( $\sigma$ )	OR( $\sigma_1$ )	"

**Mencish** with uniform, unique and context-independent syntax

$\psi(x)$	$\phi(x)$	$\sigma_1$	<b>unary-formula</b> with <b>variable</b> $\sigma_1$
$\psi(x,y)$	$\phi(x,y)$	$\sigma_1$	<b>binary-formula</b> with <b>variable strings</b> $\sigma_1 \sigma_2$
$\psi(x,y)$	$\phi(x,y)$	$\sigma_1$	<b>multary-formula</b> with <b>variable strings</b> $\sigma_1 \sigma_2 \sigma_3 \dots$
<i>weird construction:</i>			
$A(x,y) = \{z \in y : \phi(x,z)\}$		$\sigma_a$	$\forall \sigma_3 [ [\sigma_3 \in \sigma_a] \leftrightarrow [ [(\sigma_3 \in \sigma_2) \wedge [\sigma_1]] ] ]$ , <b><math>\sigma_1</math> formula</b> with $\sigma_1 \sigma_3$

**section 2.4 Interpretations** of 'Axiomatic Set Theory' *ast-web.pdf* by Peter Koepke, Alex Wilke, R. Knight 2006, Boris Zilber Oxford; and similar sources introduce further symbol combinations, e.g. :

collection of sentences *S* of sentence  $\sigma$ , *S* is a model of  $\sigma$

$\langle M, E \rangle$  domain *M*, binary relation *E*

$\langle M, \in \rangle$

$\vdash$	$\models$		
$\Phi'$	$\Phi \cup$	$\Phi^\Phi$	$\Phi^U$
$\text{card}(\mathbf{R})$	$\text{card}(\mathbf{N})$	$\mathbf{Q}$	

And its getting worse. Jürgen-Michael Glubrecht, Arnold Oberschelp, Günter Todt: *Klassenlogik*. On pages 449-455 they list symbols, 2 pages nonalphabetic and 5 pages relating to some sort of alphabets.



John Bercow: Order !

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### Johann Wolfgang von Goethe, *Faust Part I, Scene IV The Study*

**Student** After all that, I feel as stupid  
As if I'd a mill wheel in my head.

Mir wird von alledem so dumm,  
Als ging, mir ein Mühlrad im Kopf herum

**Mephistopheles** Next, before all else, you'll fix  
Your mind on Metaphysics!  
See that you're profoundly trained  
In what never stirs in a human brain:  
You'll learn a splendid word  
For what's occurred or not occurred.

Nachher, vor allen andern Sachen,  
Müßt Ihr Euch an die Metaphysik machen!  
Da seht, daß Ihr tiefsinnig faßt,  
Was in des Menschen Hirn nicht paßt;  
Für was drein geht und nicht drein geht,  
Ein prächtig Wort zu Diensten steht.

To me, set theory and the axiom,  
Is both a mill and a conundrom.

Die Lehre von Mengen mit Axiomen,  
Verwirrt meine Sinne, oh schrecklich Omen.